

achieve a high degree of confidence in the evaluated upper limits, we used for the computation of  $A$  the net number of counts (that is, target minus background) recorded in  $AB\bar{H}\bar{M}\bar{L}$ , plus *three times* the standard error in this net number.

Following a suggestion by C. Werntz,<sup>10</sup> we have calculated a lower limit of the half-life of H<sup>4</sup>, assuming a modest production cross section  $\sigma_p = 0.1 \mu\text{b}$ . Our result for this lower limit is 2 years.

It may be noticed here that our setup, especially counter  $B$ , is also sensitive to gamma rays—but with a smaller efficiency. For instance,  $\eta > 4\%$  for 2½-MeV gamma's in counter  $B$ .

<sup>10</sup> C. Werntz (private communication).

#### IV. CONCLUSION

The upper limits of the production cross section of H<sup>4</sup> from lithium are of the order of  $2 \times 10^{-4} \mu\text{b}$ , many orders of magnitude smaller than the cross sections of similar process in other elements, as shown in Table II. We therefore conclude that it is very unlikely that H<sup>4</sup> is a  $\beta$  emitter with a half-life ranging from 1 min to 2 years. It is also unlikely that H<sup>4</sup> should have a half-life longer than 2 years because that would give a  $\log ft \geq 14$  for its decay. It is clear that new decay schemes of a long-lived H<sup>4</sup> not discussed here (for instance, one that involves a hitherto undiscovered level in He<sup>4</sup>) are also unlikely when we consider that no delayed gamma rays were observed.

### Alpha-Particle Resonance\*

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A phase-shift analysis of the elastic  $p$ - $t$  cross section in the energy range 0.1 to 0.76 MeV has been made. It is shown that the low peak observed at 0.3 MeV is consistent with a  $0^+$  resonance at  $0.5 \pm 0.1$  MeV. The shape of the observed cusp in the  $p$ - $t$  elastic cross section at the threshold energy of the  $n$ -He<sup>3</sup> channel is used to demonstrate that the energy behavior of the singlet and triplet  $s$ -wave phase obtained must be qualitatively unique. A direct comparison is made of the Breit-Wigner single-level approximation and the alternative two-channel scattering length description of the state.

#### I. INTRODUCTION

A PEAK has been observed in the energy spectrum of breakup neutrons from the reaction  $t(d,np)t$  in two separate high-resolution experiments.<sup>1,2</sup> The peak in the neutron distribution corresponds to an energy of 0.5 in the center of mass of the  $t$ - $p$  system. The peak is most pronounced in the forward direction, disappearing entirely at about  $60^\circ$  in the laboratory. Such a distribution is typical of a stripping process in which the nucleon which interacts most strongly with the target nucleus interacts principally in an  $S$  state. The shape of the neutron peak can be fit fairly well<sup>3</sup> by assuming an  $S$ -wave resonance in the  $t$ - $p$  system at an energy of about 0.5 MeV. The excitation energy of this state is 20.4 MeV with respect to the He<sup>4</sup> ground state.

However, as Watson has shown,<sup>4</sup> in a three-body breakup one can expect peaks in the spectrum of one of the fragments even when the important phase shift of the remaining two-body system does not have a resonance

behavior. The only requirement is that this phase shift be a rapidly varying function of energy. Near a threshold this rapid variation can be due to a large scattering length. For example, a distinct peak is observed in the neutron spectrum from  $d(p,pp)n$  due to the strong singlet  $S$ -wave interaction of the two protons. In order to determine whether one of the  $S$ -wave phase shifts actually passes through  $90^\circ$ , we have undertaken a phase-shift analysis of the differential cross section for elastic  $t$ - $p$  scattering which has been recently measured by Jarmie *et al.*<sup>5</sup> in the energy region 0.16 to 0.52 MeV.

The cross section was measured at a single angle of  $120^\circ$  in the center of mass. A low peak appears at about 0.3 MeV, which corresponds to the peak observed in the  $t(d,np)t$  reaction. In the elastic scattering it is shifted to slightly lower energy because of the different energy dependence of the kinematic factors in the two-body elastic scattering as compared to the three-body final state. The presence of a large Coulomb amplitude in the elastic scattering also has the effect of making the peak less prominent. In the sections which follow the attempt to obtain the  $^1S_0$  and  $^3S_1$  phase shifts is discussed. For both  $S$ -wave phase shifts the effect of the closed  $n$ -He<sup>3</sup>

\* Work partially supported by the U. S. Air Force Office of Scientific Research.

<sup>1</sup> H. W. Lefevre, R. R. Borchers, and C. H. Poppe, *Phys. Rev.* **128**, 1328 (1962).

<sup>2</sup> C. H. Poppe, C. H. Holbrow, and R. R. Borchers, *Phys. Rev.* **129**, 733 (1963).

<sup>3</sup> Carl Werntz, *Phys. Rev.* **128**, 1336 (1962).

<sup>4</sup> K. M. Watson, *Phys. Rev.* **88**, 1163 (1952).

<sup>5</sup> Nelson Jarmie, M. G. Silbert, D. B. Smith, and J. S. Loos, *Phys. Rev.* **130**, 1987 (1963).

channel and the coupling of the open  $p$ - $l$  channel to it have been taken into account by using a  $2 \times 2$  transition matrix for each spin and parity partial wave. Satisfactory fits to the differential cross section as well as to the  $\text{He}^3(n,n)\text{He}^3$  and  $\text{He}^3(n,p)l$  cross sections for thermal neutrons are obtained by assuming a broad  ${}^1S_0$  resonance below the  $n$ - $\text{He}^3$  threshold.

## II. PHASE-SHIFT PARAMETRIZATION

We are interested in center-of-mass energies in the range 0.1 to 0.76 MeV. At such low energies the singlet and triplet  $S$ -wave phase shifts should be the most important. For the sake of completeness we write down the  $S$ -wave contribution to the differential cross section in terms of the phase shifts.<sup>6</sup>

$$\frac{d\sigma^S}{d\Omega}(\theta, E_p) = \frac{1}{4k_p^2} \{ [\sin^2\delta_1 + 3\sin^2\delta_3] - \eta \csc^2\frac{1}{2}\theta [\sin\delta_1 \cos(\delta_1 + 2\eta \ln \sin\frac{1}{2}\theta) + 3\sin\delta_3 \cos(\delta_3 + 2\eta \ln \sin\frac{1}{2}\theta)] + \eta^2 \csc^4\frac{1}{2}\theta \}. \quad (1a)$$

The singlet phase shift is represented by  $\delta_1$  and the triplet by  $\delta_3$ . The proton wave number  $k_p$  is defined by  $k_p = [3mE_p/2\hbar^2]^{1/2}$  and the Coulomb parameter  $\eta$  is related to  $k_p$  by the equation  $\eta = m e^2 / \hbar k_p$ . Since use will be made of the  $n$ - $\text{He}^3$  total cross sections for thermal neutrons we also write down the elastic and inelastic cross sections in terms of the appropriate  $T$ -matrix elements.

$$\sigma_{nn} = \frac{\pi}{k_n^2} [ |T_{nn}^1|^2 + 3 |T_{nn}^3|^2 ], \quad (1b)$$

$$\sigma_{np} = \frac{\pi}{k_n^2} [ |T_{np}^1|^2 + 3 |T_{np}^3|^2 ]. \quad (1c)$$

The neutron wave number is defined by

$$k_n = [3m(E_p - E_t)/2\hbar^2]^{1/2}.$$

The singlet and triplet phase shifts are related to the diagonal  $T$ -matrix elements  $T_{pp}^1$  and  $T_{pp}^3$  through the equations

$$e^{i\delta_1} \sin\delta_1 = T_{pp}^1, \quad e^{i\delta_3} \sin\delta_3 = T_{pp}^3. \quad (2)$$

The height of the peak definitely rules out a resonance in the  ${}^3S_1$  partial wave. We thus assume that the resonance occurs in the singlet state. Upon making this assumption one is forced to take  $\delta_3$  to be small and negative in order to fit the curve at very low energies where the interference between the Coulomb and nuclear amplitudes is large. This is in agreement with the single-channel phase-shift analysis of Frank and Gammel<sup>7</sup> in which the triplet  $S$ -wave phase shift was found to be small and negative and the singlet

large and positive in the energy region above the  $n$ - $\text{He}^3$  threshold.

## ${}^1S_0$ Phase Shift

The energy dependence of the phase shift  $\delta_1$  is taken to be given by a Breit-Wigner single-level approximation

$$\delta_1 = \alpha + \xi, \quad \alpha = \tan^{-1} \frac{-F_0(k_p a)}{G_0(k_p a)}, \quad (3)$$

$$\xi = \tan^{-1} \frac{\Gamma_p}{E_0 + \Delta_n + \Delta_p - E_p}, \quad \Gamma_p = P_p^+(E_p) \gamma_p^2,$$

$$\Delta_n = \gamma_n^2 [B_n - S_n^-(E_n)], \quad \Delta_p = \gamma_p^2 [B_p - S_p^+(E_p)].$$

These equations are valid only for negative values of the neutron energy,  $E_n$ . In terms of the threshold energy for  $t(p,n)\text{He}^3$ ,  $E_t$ ,  $E_n = E_p - E_t$ . The parameters describing the resonance are the resonant energy  $E_0$ , the reduced neutron and proton widths  $\gamma_n^2$  and  $\gamma_p^2$ , and the channel radius  $a$ . The energy-dependent functions  $P_p^+(E_p)$ ,  $S_p^+(E_p)$ , and  $S_n^-(E_n)$  are listed in Ref. 6. The constants  $B_n$  and  $B_p$  are the logarithmic derivatives of the channel wave functions on the nuclear surface evaluated for  $E_p = E_0$ . We choose them such that  $\delta_1(E_0) = 90^\circ$ ;

$$B_p = k_p a \left. \frac{G_0'(k_p a)}{G_0(k_p a)} \right|_{E_0}, \quad B_n = -K_n a |_{E_0}, \quad (4)$$

$$K_n = [3m(E_t - E_p)/2\hbar^2]^{1/2}.$$

## ${}^3S_1$ Phase Shift

A two-channel scattering-length approximation is used for the triplet shift. Ross and Shaw<sup>8</sup> show that the energy dependence of the multichannel scattering matrix  $T$  can be expressed most conveniently through the introduction of an  $M$  matrix defined by the relation (for  $S$  waves, all channels unchanged)

$$T = k^{\frac{1}{2}} (M - ik)^{-1} k^{1/2}. \quad (5)$$

The  $M$  matrix has an effective-range-type expansion

$$M(E) = M(E_1) + \frac{1}{2} R [k^2 - k^2(E_1)]. \quad (6)$$

In the above equations all quantities are square matrices with the order of the matrix equal to the number of channels being considered. The elements  $k_i^{\frac{1}{2}}$  of the diagonal matrix  $k^{\frac{1}{2}}$  are the square roots of the wave numbers in the various open channels.  $M(E_1)$  and  $R$  are constant matrices, the latter being approximately diagonal,  $R_{ij} \cong 0$ ,  $i \neq j$ . For a charged channel one can show that Eqs. (5) and (6) are valid if one makes the replacement<sup>9</sup>

$$k_c \rightarrow C_0^2 k_c, \quad M(E_1) \rightarrow M(E_1) - g(\eta_c)/D, \quad (7)$$

$$C_0^2 = 2\pi\eta_c / (e^{2\pi\eta_c} - 1).$$

<sup>6</sup> A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

<sup>7</sup> R. M. Frank and J. L. Gammel, Phys. Rev. **99**, 1405 (1955).

<sup>8</sup> Marc Ross and Gordon Shaw, Ann. Phys. (N. Y.) **13**, 147 (1961).

<sup>9</sup> R. G. Sachs, Nuclear Theory (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953), p. 126.

TABLE I. Sets of values of the parameters which yield satisfactory fits to the experimental  $p$ - $l$  cross section.

$a$ ( $10^{-13}$ cm)	$E_0$ (MeV)	$\gamma_n^2$ (MeV)	$\gamma_p^2$ (MeV)	$\alpha$ ( $10^{12}$ cm $^{-1}$ )	$\beta^2$ ( $10^{24}$ cm $^{-2}$ )	$\gamma$ ( $10^{12}$ cm $^{-1}$ )	$\frac{d\sigma^P}{d\Omega}(120^\circ, E_t)$ (mb/sr)
3.0	0.4	1.50	4.18	-3.06	+4.59	+0.94	91
3.0	0.5	0.50	4.44	-2.08	+3.52	+0.90	80
3.0	0.6	0.50	4.77	-1.57	+2.31	+1.14	57
3.6	0.4	3.00	3.02	-3.97	+3.39	+2.04	90
3.6	0.5	3.00	3.84	-2.66	+2.91	+1.69	76
3.6	0.6	2.00	3.52	-1.82	+2.23	+1.94	53
4.2	0.4	2.09	2.09	-2.87	+5.21	+1.08	94
4.2	0.5	1.70	2.40	-2.60	+3.24	+1.49	76
4.2	0.6	1.57	2.60	-1.90	+1.81	+1.77	53

The length  $D=1/\eta_c k_c$  and the functional form of  $g(\eta_c)$  is given in Ref. 9. The function  $-g(\eta)$  is a decreasing function of energy whereas, for positive effective range,  $R_{ii}[k^2-k^2(E_1)]$  is an increasing function of energy. We have estimated that the changes in these two functions are of the same order in the energy range 0.1 to 0.76 MeV. Therefore, we take the scattering-length approximation to be that of choosing the  $M$  matrix to be a constant matrix;

$$M = \begin{pmatrix} \alpha & \beta \\ \beta & \gamma \end{pmatrix}. \quad (8)$$

In terms of the matrix elements  $\alpha$ ,  $\beta$ , and  $\gamma$  the three triplet  $T$ -matrix elements are

$$\begin{aligned} T_{pp}^3 &= C_0^2 k_p (\gamma - i k_n) d^{-1}, \\ T_{np}^3 &= T_{pn}^3 = C_0 k_p^{1/2} k_n^{1/2} \beta d^{-1}, \\ T_{nn}^3 &= k_n (\alpha - i C_0^2 k_p) d^{-1}, \\ d &= \|M - i k\|. \end{aligned} \quad (9)$$

One sees from Eqs. (1b) and (1c) that  $|T_{nn}^3|^2$  and  $|T_{np}^3|^2$  are defined only for  $E_n \geq 0$ . However, the representation given above for  $T_{pp}^3$  can be analytically continued below  $E_t$  by making the replacement  $k_n \rightarrow i K_n$ . The expansion of  $\delta_3$  in terms of  $\alpha$ ,  $\beta$ , and  $\gamma$  can be obtained from Eq. (9). It is

$$C_0^2 k_p \cot \delta_3 = \alpha - \beta^2 / (\gamma + K_n). \quad (10)$$

### $p$ -Wave Phase Shifts

If no spin-orbit splitting occurs, only two  $p$ -wave phase shifts, the singlet and triplet, are required. If  $\delta_1$  is fixed at  $E_p = E_t$  both  $p$ -wave phase shifts as well as  $\delta_3$  can be obtained from the angular dependence<sup>10</sup> of  $d\sigma/d\Omega(\theta, E_t)$ . The contributions of these three phase shifts for  $E_p < E_t$  can be estimated from a scattering length approximation. This calculation was actually carried out for a number of choices of the resonance parameters and in every case the calculated cross section was always much too high at  $E_p = E_0$ . The reason seems

to be that in assuming no spin-orbit splitting the entire isotropic contribution must come from  $\sin^2 \delta_1 + 3 \sin^2 \delta_3$ . This makes  $\delta_3$  come out to be equal to, roughly,  $-25^\circ$  at  $E_t$ . Upon extrapolating down to  $E_0$  the contribution of  $\delta_3$  to the cross section is too large. If the  $p$  waves contribute to the isotropic term, their contribution falls off much more rapidly with energy. The phase shift  $\delta_3$  can then be chosen to be smaller at  $E_t$  and the experimental cross section can be fit at  $E_0$  since the  $p$  waves contribute very little at this lower energy. Since one must determine five  $p$ -wave  $T$ -matrix elements in the presence of spin-orbit splitting instead of two a unique determination of these elements is out of the question. Therefore, the drastic assumption was made that the contributions of the  $p$  waves to  $d\sigma/d\Omega(120^\circ, E_p)$  for  $E_p < E_t$  is proportional to  $E_p^2$ . Including this approximation, the complete expression which was used for the differential cross section becomes

$$\frac{d\sigma}{d\Omega}(120^\circ, E_p) = \frac{d\sigma^S}{d\Omega}(120^\circ, E_p) + \frac{E_p^2}{E_t^2} \frac{d\sigma^P}{d\Omega}(120^\circ, E_t). \quad (11)$$

### III. NUMERICAL FIT

Altogether 8 parameters are at our disposal to obtain a good fit. These are  $E_0$ ,  $\gamma_p^2$ ,  $\gamma_n^2$ ,  $a$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $d\sigma^P/d\Omega(120^\circ, E_t)$ . It turns out that if only five experimental quantities are matched one obtains a good fit at all energies. These quantities are:

$$\frac{d\sigma}{d\Omega}(120^\circ, 0.2 \text{ MeV}) = 210 \text{ mb/sr},$$

$$\frac{d\sigma}{d\Omega}(120^\circ, E_t) = 192 \text{ mb/sr},$$

and

$$\frac{d\sigma}{d\Omega}(120^\circ, E_0) \quad \text{which depends on } E_0; \quad (12)$$

also<sup>11</sup>

$$\sigma_{nn} = 1.8 \text{ b},$$

$$\sigma_{np} = 5280 \text{ b}.$$

<sup>10</sup> M. E. Ennis and A. Hemmendinger, Phys. Rev. **95**, 772 (1954).

<sup>11</sup> J. D. Seagrave, L. Cranberg, and J. E. Simmons, Phys. Rev. **119**, 1981 (1960).

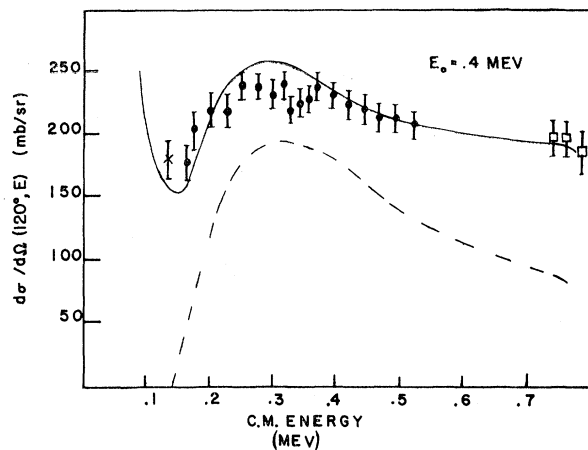


FIG. 1.  $p$ - $t$  differential cross section at  $120^\circ$  calculated from the parameters of Table I listed for channel radius  $a=4.2$  F and resonance energy  $E_0=0.4$  MeV. The dashed curve shows the contribution of the singlet  $s$ -wave amplitude to the cross section. The data points have been reproduced from Ref. 5.

In our numerical fitting  $E_0$ ,  $a$ , and  $\gamma_n^2$  were treated as independent parameters and the other five parameters determined for each set of values of the independent ones. Poppe<sup>2</sup> *et al.* found that  $E_0 \cong 0.5$  MeV. In making our fits we selected three different values for  $E_0$  so as to bracket this value. They are 0.4, 0.5, and 0.6 MeV. (For  $E_0$  much lower than 0.4 MeV the contribution of the resonance to the cross section is higher than the observed value.) Three different values of the channel radius,  $a$ , were also employed,  $a=3.0$ , 3.6, and 4.2 F. The range of values possible for  $\gamma_n^2$  was limited by two considerations. Firstly, the sum of the two reduced widths must be less than the Wigner limit,  $\gamma_n^2 + \gamma_p^2 < 3\hbar^2/2ma^2$ . Secondly, since the resonant state should be one of definite isobaric spin  $\gamma_n^2 \cong \gamma_p^2$ . Both of these conditions could be best satisfied for  $a=4.2$  F. For this reason it is

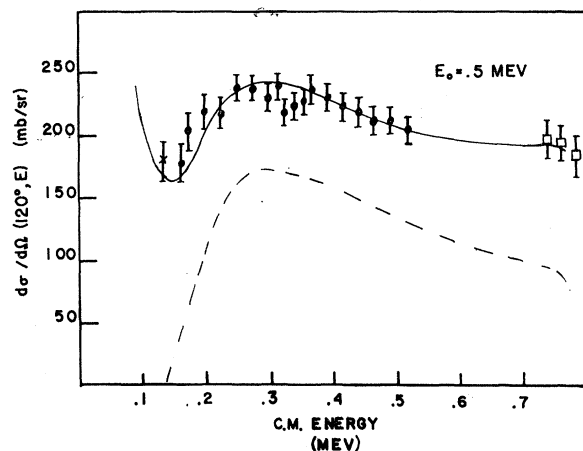


FIG. 2.  $p$ - $t$  differential cross section at  $120^\circ$  calculated from the parameters of Table I listed for channel radius  $a=4.2$  F and for resonance energy  $E_0=0.5$  MeV. The dashed curve shows the contribution of the singlet  $s$ -wave amplitude to the cross section.

believed that the parameters obtained with this radius are the most reliable.

Table I contains sets of values of the eight parameters which give satisfactory fits. In most cases  $\gamma_n^2$  was selected so that the ratio  $\gamma_n^2/\gamma_p^2$  is as near to one as possible under the constraint that the sum of the widths be less than the Wigner limit. The three curves obtained for  $a=4.2$  F are displayed in Figs. 1-3. The curves for the two smaller values of the channel radius are virtually indistinguishable from the curves shown.

#### IV. DISCUSSION

It is clear that we have obtained a reasonable fit to  $d\sigma/d\Omega(120^\circ, E_p)$  as well as reproducing the values of  $\sigma_{nn}$  and  $\sigma_{np}$  by assuming that a  $0^+$  resonance of the  $\alpha$  particle occurs in the energy region between the  $t$ - $p$  and  $n$ - $\text{He}^3$  thresholds. Because of the crude approximation made

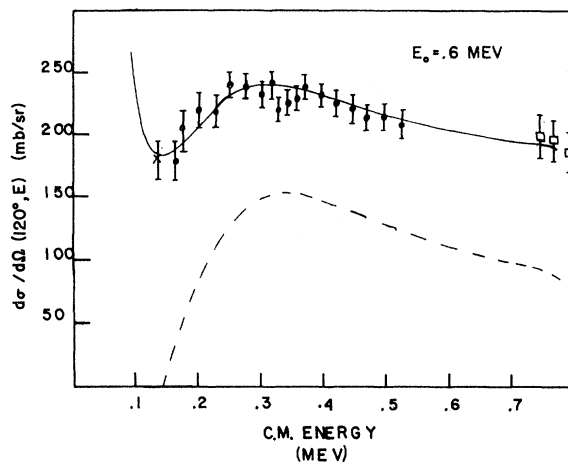


FIG. 3.  $p$ - $t$  differential cross section at  $120^\circ$  calculated from the parameters of Table I listed for channel radius  $a=4.2$  F and for resonance energy  $E_0=0.6$  MeV. The dashed curve shows the contribution of the singlet  $s$ -wave amplitude to the cross section.

for the  $p$  waves the reduced widths and the resonant energy itself are only approximately determined. A more accurate phase-shift fit, one which is independent of assumptions regarding the energy behavior of the phase shifts, can be made only if the low-energy differential cross section is measured at other angles. A very accurate analysis at such low energy would be possible because of the large interference between the  $s$ -wave phase shifts and the Coulomb amplitude. The question naturally arises of whether our fit is qualitatively unique, that is, must one necessarily assume that the singlet phase shift passes through  $90^\circ$  at some energy below the  $n$ - $\text{He}^3$  threshold. We think that the observed cusp<sup>12</sup> in the  $t$ - $p$  elastic cross section in the vicinity of the  $n$ - $\text{He}^3$  threshold does force one to this assumption.

If there were a single  $s$ -wave phase shift in the  $t$ - $p$

<sup>12</sup> Nelson Jarmie and Robert L. Allen, Phys. Rev. 114, 176 (1959).

system there would be a particularly simple relationship<sup>13</sup> between the phase shift and the ratio of the energy derivatives of the differential cross section above and below  $E_t$ . This relationship is

$$\lim_{\Delta E \rightarrow 0} \frac{d}{dE_p} \frac{d\sigma}{d\Omega}(\theta, E_t + \Delta E) / \frac{d}{dE_p} \frac{d\sigma}{d\Omega}(\theta, E_t - \Delta E) = -\tan \delta. \quad (13)$$

Since the experimental resolution attained in Ref. 12

was of the order of 10 keV one can only estimate the required derivatives for  $\Delta E > 10$  keV. From the curve for  $\theta_{o.m.} = 59^\circ$  and for  $\Delta E = 20$  keV we can obtain the estimate of  $\sim +3$  for the required ratio. This corresponds to a value for  $\delta$  of  $+108^\circ$ . Equation (13) can be generalized to the actual case where there are two  $s$ -wave phase shifts as well as a large Coulomb amplitude which interferes with both  $s$  waves. The result, which is obtained straightforwardly from the representation of the requisite  $T$ -matrix elements given in Eq. (9) is

$$\begin{aligned} \lim_{\Delta E \rightarrow 0} \frac{\frac{d}{dE_p} \frac{d\sigma}{d\Omega}(\theta, E_t + \Delta E)}{\frac{d}{dE_p} \frac{d\sigma}{d\Omega}(\theta, E_t - \Delta E)} = & - \left\{ |T_{np^1}|^2 \left[ \sin^2 \delta_1 - \frac{\eta}{2} \operatorname{csc}^2 \frac{\theta}{2} \sin \left( 2\delta_1 + 2\eta \ln \sin \frac{\theta}{2} \right) \right] \right. \\ & \left. + 3 |T_{np^3}|^2 \left[ \sin^2 \delta_3 - \frac{\eta}{2} \operatorname{csc}^2 \frac{\theta}{2} \sin \left( 2\delta_3 + 2\eta \ln \sin \frac{\theta}{2} \right) \right] \right\} \\ & \times \left\{ |T_{np^1}|^2 \left[ \sin \delta_1 \cos \delta_1 - \frac{\eta}{2} \operatorname{csc} \frac{\theta}{2} \cos \left( 2\delta_1 + 2\eta \ln \sin \frac{\theta}{2} \right) \right] \right. \\ & \left. + 3 |T_{np^3}|^2 \left[ \sin \delta_3 \cos \delta_3 - \frac{\eta}{2} \operatorname{csc} \frac{\theta}{2} \cos \left( 2\delta_3 + 2\eta \ln \sin \frac{\theta}{2} \right) \right] \right\}^{-1}. \quad (14) \end{aligned}$$

Upon inserting the values of the phase shifts and the inelastic  $T$ -matrix elements obtained with  $a = 4.2$  F into the right side of Eq. (14), the values of the ratio are obtained which are listed in Table II. The qualitative agreement is evident.

The main point of this analysis is that  $\delta_1$  must be slightly larger than  $90^\circ$  at  $E_t$ . If one assumes that it starts from  $0^\circ$  and increases, it clearly must pass through  $90^\circ$ ; a behavior similar to the singlet  $n$ - $p$  phase shift is ruled out. The other possibility is that it starts from  $180^\circ$  and decreases rapidly to a value slightly greater than  $90^\circ$  at  $E_t$ . Such a behavior could only occur if there were a bound  $0^+$  state slightly below the  $t$ - $p$  threshold. Since no such state was observed in the  $t(d, np)t$  experiments, we must conclude that  $\delta_1$  does pass through  $90^\circ$ .

Next, we would like to examine a method of description of the resonance which is an alternative to the Breit-

Wigner formula. Dalitz and Tuan<sup>14</sup> have also developed an effective-range-type expansion for analyzing multi-channel scattering. They have pointed out that a resonance of the type with which we are dealing in this paper—a resonance which is between the thresholds for two different channels—can be described by a scattering length or a “zero-range” approximation to the  $K$  matrix (the inverse of the  $M$  matrix). Reference to Eq. (10) of this paper makes it clear that if  $\alpha$ ,  $\beta$ , and  $\gamma$  are such that

$$\alpha - \frac{\beta^2}{\gamma + K_n} \begin{cases} < 0 & E_p > E_p' \\ > 0 & E_p < E_p' \end{cases}, \quad (15)$$

for some  $E_p'$  between the two thresholds  $\delta_1$  passes rapidly through  $90^\circ$ .

We have compared the behavior of  $\delta_1$  when computed from a Breit-Wigner resonance formula [Eq. (3)] to its behavior when computed from a constant  $M$  matrix [Eq. (8)]. In each case there are three constants to be determined. We have chosen the set of resonance parameters  $E_0 = 0.5$  MeV,  $\gamma_n^2 = 1.7$  MeV, and  $\gamma_p^2 = 2.4$  MeV as the basis for the comparison. Then the constants  $\alpha$ ,  $\beta$ , and  $\gamma$  are chosen so that  $\delta_1 = 90^\circ$  at  $E_p = 0.5$  MeV and so that the two quantities  $|T_{nn^1}|^2/k_n^2$  and  $|T_{np^1}|^2/k_n^2$  are the same in each case for  $E_n = E_t$ . The values of the phase shifts obtained by each method over the range of interest of the energy are listed in Table III.

TABLE II. The limit, as  $E_p \rightarrow E_t^\pm$ , of the ratio of the energy derivative of the  $p$ - $t$  differential cross section above the  $n$ -He<sup>3</sup> threshold to the energy derivative of the cross section below threshold. The ratio has been calculated for  $\theta = 60^\circ$  using the three sets of parameters in Table I listed under  $a = 4.2$  F. The “observed” value estimated from Fig. 1 of Ref. 12 is  $+3$ .  $E_0$  is given in MeV.

$E_0 =$	0.4	0.5	0.6
$a/b =$	+2.31	+4.45	+6.48

<sup>13</sup> R. G. Newton, Ann. Phys. (N. Y.) 4, 29 (1958).

<sup>14</sup> R. H. Dalitz and S. F. Tuan, Ann. Phys. (Paris) 3, 307 (1960).

TABLE III. A comparison of the  ${}^1S_0$  phase shift calculated from a Breit-Wigner single level formula to that calculated from a two-channel scattering length approximation.

C.m. energy MeV	$\delta_1$ (B.W.) degrees	$\delta_1$ (S.L.) degrees
0.20	47	55
0.50	90	90
0.76	105	109

There is a significant difference at the lowest energy. However, since the actual value of the phase shift is not known at each energy one can not choose one three-parameter fit over the other. Poorer agreement resulted from including the function  $g(\eta)$  in the  $M$  matrix.

We now turn to the question of the assignment of isobaric spin to the resonance. A nucleon and an  $A=3$  nucleus in a given spin and parity state can be coupled to a total isobaric spin of either 0 or 1. A mixture of the two is possible only if a  $T=0$  and a  $T=1$  state of the four nucleon system with similar boundary conditions lie very close together in energy. In view of the singular lack of profusion of  $\alpha$ -particle states this is very unlikely. An assignment of  $T=1$  to the state would force one to the conclusion<sup>15</sup> that a very weakly bound  $H^4$  with a  $0^+$  ground state exists.  $H^4$  would decay via  $\beta^-$  emission to the ground state of  $He^4$  with a long lifetime of the order of an hour. A number of experiments<sup>16-18</sup> been carried out in the past in which an  $H^4$  with a lifetime of the order of several minutes or less could have been detected if it existed. In all cases no activity attributable to  $H^4$  was observed. Very recently, Spicer<sup>19</sup> and Nefkens and Moscati<sup>20</sup> have exposed lithium targets to gamma rays for times as long as several hours in order to detect the presence of an  $H^4$  with a lifetime appropriate to a  $0^+$  state. They give upper limits for the cross section for production of  $0.6 \mu b$  and  $2 \times 10^{-4} \mu b$ , respectively. Since the latter figure is many orders of magnitude lower than what can be reasonably expected the experimental evidence strongly favors a choice of  $T=0$ .

<sup>15</sup> C. Wernitz and J. G. Brennan, Phys. Letters **6**, 113 (1963).

<sup>16</sup> L. Rosen and J. E. Brolley, Jr., Phys. Rev. **117**, 1307 (1960).

<sup>17</sup> H. A. Grench, W. L. Imhof, and F. J. Vaughn, Bull. Am. Phys. Soc. **7**, 268 (1962); AFSWC-TDR-62-26, 1962, Air Force project No. 8802 (unpublished).

<sup>18</sup> Kenneth G. McNeill and Waldo Rall, Phys. Rev. **83**, 1244 (1951).

<sup>19</sup> B. M. Spicer, Phys. Letters **6**, 88 (1963).

<sup>20</sup> B. M. K. Nefkens and G. Moscati (to be published).

Such a choice is also consistent with the shell model. Elliot and Skyrme<sup>21</sup> have shown that the first excited state with the spatial partition [4] must be a  $2\hbar\omega$ , even parity state. Exciting a single nucleon into the  $1p$  shell while retaining a totally symmetric space state leaves the  $He^4$  in its ground state and changes only the state of the spurious center-of-mass motion introduced by using shell-model wave functions. A  $T=0$ ,  $S=0$  state of the four nucleon system which has a big probability for breaking into either  $t-p$  or  $He^3-n$  would be expected to have a spatial wave function that transforms like [4]. The only difficulty with this is that if one takes the observed rms radius of the alpha and computes the corresponding value of  $\hbar\omega$  one finds<sup>22</sup> that  $2\hbar\omega$  is roughly 50 MeV. This is considerably larger than the 20.4-MeV splitting of the ground and first excited state that has been observed. We are currently calculating the positions of the  $0^+$ ,  $T=0$  and  $T=1$  states starting with a reasonable two-nucleon potential.

In the preceding discussion it has been assumed that the resonance in question is a pure isobaric spin state. Baz<sup>23</sup> has pointed out that there are many instances in light nuclei of resonances occurring very near the threshold for a new channel. These "threshold resonances" are characterized by having reduced widths for breakup into the new channel which are large compared to the other channel widths. This implies, in some cases, that a significant mixing of isobaric spin states is occurring. This mixing apparently does not occur in the state in question since it lies equally close to the two important channels. In fact, one can always find an energy  $E_p'$ ,  $0 < E_p' < 0.76$  MeV, such that  $B_n = B_p$  for any reasonable choice of the channel radii. This indicates that the wave functions are very similar in the two channels even in the asymptotic region.

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<sup>21</sup> J. P. Elliot and T. H. R. Skyrme, Proc. Roy. Soc. (London) **A232**, 561 (1955).

<sup>22</sup> Paul Goldhammer, Rev. Mod. Phys. **35**, 40 (1963).

<sup>23</sup> A. S. Baz', in *Advances in Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1959), Vol. 8, p. 349.